# Exam 2 Review

MA 265

March 2021

**1.** A square matrix $A$ is said to be **symmetric** if $A = A^t$. Show that the set of $2 \times 2$ symmetric matrices is a subspace of $M_{2\times 2}$. Find a basis for this subspace.

**2.** A square matrix $A$ is said to be **skew-symmetric** if $A = -A^t$. Show that the set of $2 \times 2$ skew-symmetric matrices is a subspace of $M_{2\times 2}$. Find a basis for this subspace.

**3.** Consider the subset of $\mathbb{P}_2$ consisting of all polynomials $p(t)$ with $p(0) = 0$. Show this is a subspace of $\mathbb{P}_2$ and find a basis for it. Is the subset of polynomials $p(t)$ with $p(0) = 1$ a subspace? Why or why not?

**4.** Consider the subset of $\mathbb{P}_2$ consisting of all polynomials $p(t)$ with $p(2) = 0$. Show this is a subspace of $\mathbb{P}_2$ and find a basis for it. Is the subset of polynomials $p(t)$ with $p(1) = 2$ a subspace? Why or why not?

**5.** Consider the subset of $\mathbb{P}_3$ consisting of all polynomials $p(t)$ with $p(1) = p(-1)$. Is this a subspace of $\mathbb{P}_3$? If yes, produce a basis for it and if not explain why it's not a subspace.

**6.** Consider the map $T : \mathbb{R}^2 \to \mathbb{P}_1$ given by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = at+b$. Is this a linear transformation? Why or why not?

**7.** Consider the map $T : M_{2\times 2} \to \mathbb{R}^1$ given by $T(A) = \det A$. Is this a linear transformation? Why or why not?

**8.** Consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ given by $T\big(p(t)\big) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$. Find bases for the kernel and range of $T$.

**9.** Consider the linear transformation $T : \mathbb{P}_3 \to M_{2\times 2}$ given by $T\big(p(t)\big) = \begin{bmatrix} p(0) & p(1) \\ p(1) & 0 \end{bmatrix}$. Find bases for the kernel and range of $T$.

**10.** Consider the linear transformation $T : M_{2\times 2} \to M_{2\times 2}$ given by $T(A) = A - A^t$. Find bases for the kernel and range of $T$.

**11.** In $\mathbb{P}_2$, is $\{t^2+1, t-2, t+1\}$ linearly independent? Why or why not?

**12.** In $M_{2\times 2}$, is $\left\{ \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} \right\}$ linearly independent? Why or why not?

**13.** Find the dimensions of each subspace, kernel, and range in all of the previous problems.

**14.** If $T : \mathbb{P}_3 \to M_{2\times 2}$ is a linear transformation and $\operatorname{range} T = M_{2\times 2}$ what can you say about the kernel of $T$?

**15.** If $T : M_{2\times 2} \to M_{2\times 2}$ is a linear transformation and $\dim\ker T = 1$ what can you say about the range of $T$?

**16.** If $T : \mathbb{R}^4 \to \mathbb{P}_2$ is a linear transformation and $\dim\ker T = 1$ what can you say about the range of $T$?

**17.** Find bases for the column space and row space of

$$\begin{bmatrix} 3 & 4 & -3 \\ 2 & 5 & 5 \\ -2 & -3 & 1 \end{bmatrix}$$

**18.** Find bases for the column space and row space of

$$A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

**19.** Find bases for the column space and row space of

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 2 & -2 \\ 2 & 1 & -3 \end{bmatrix}$$

**20.** Given the matrix

$$A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$$

(a) Find the characteristic polynomial of $A$.

(b) Find the eigenvalues of $A$.

(c) Find the corresponding eigenvectors to the eigenvalues of $A$.

(d) Find the dimensions of the eigenspaces of $A$ for each eigenvalue.

(e) Is $A$ diagonalizable? If yes, diagonalize $A$. If not, explain why.

**21.** Repeat (a)-(e) above with

$$A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$$

**22.** Repeat (a)-(e) above with

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$$

**23.** Repeat (a)-(e) above with

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

**24.** Repeat (a)-(e) above with

$$A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$$

**25.** Write an expression for $A^k$ for the diagonalizable matrices you found above.

**26.** Let $T : \mathbb{P}_3 \to \mathbb{P}_3$ be the linear transformation given by $T\big(p(t)\big) = p'(t) + p''(t)$. Find the matrix representation of $T$, $[T]_\beta$ where $\beta = \{1, t, t^2, t^3\}$, the standard basis of $\mathbb{P}_3$.

**27.** Let $T : M_{2\times 2} \to M_{2\times 2}$ be the linear transformation given by $T(A) = A + A^t$. Find the matrix representation of $T$, $[T]_\beta$ where $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$, the standard basis of $M_{2\times 2}$.